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Mammad SHAHMALIYEV, PhD student,
National Aviation Academy of Azerbaijan,
Xudu Mammadov str. 38, 122, Baku, AZ1129, Azerbaaijan,
mamed.shahmaliyev@gmail.com

SIMULATION MODEL OF INFINITE PERISHABLE QUEUEING INVENTORY SYSTEM WITH FEEDBACK

Perishable Queuing Inventory system with positive service time and customer feedback is considered. The system applies Variable Size Order policy for the inventory replenishment. Stochastic simulation method is used to calculate the system performance measures and find its stationary distribution. The dependence of performance measures on the reorder level is illustrated and analyzed using the numerical experiments.

Keywords: *perishable queuing-inventory system, infinite three-dimensional Markov Chain, simulation algorithm, VSO-policy, numerical experiments.*

Introduction

Queueing-Inventory System (QIS) with repeated customers was first studied in [1] and [2]. The authors of these articles independently investigated the Markov QIS model with (s, S) replenishment policy using the different approachs. When the inventory level is zero arriving initial customers (p-customers) are assumed to join the orbit of infinite length. The customers in the orbit (r-customers) repeat their calls after exponentially distributed random time. Two-dimensional Markov chain is used to describe the mathematical model of the system. Authors used the method of generating functions and Laplace transform algorithms correspondingly to calculate the stationary distribution of the system. The variation of the model with insistent r-customers and $(S-1, S)$ replenishment is considered in [3] and [4].

QIS models with positive service time and repeated customers are considered in [4—9]. Three different Markov models with single server, (s, S) replenishment policy and r-customers were studied in [4]. The generating matrix (Q-matrix) of each model was constructed and proved to have the tridiagonal form. Matrix-Analytical [10] technology was applied to calculate the performance measures and calculate the stationary distribution. In the QIS model considered in [6] the servers are treated as the inventory items and the inventory is assumed to replenish immediately. When the inventory level is zero arriving customers are assumed to join the orbit of infinite length. The corresponding three-dimensional Markov chain was constructed and the performance measures were calculated.

Markov model with finite queue of the high priority customers was studied in [7]. Inventory level does not change after servicing the low priority customer. The low priority customers are taken to the service only in case of the idle servers, otherwise they join the orbit and become impatient. The four-dimensional Markov chain was constructed to find the joint distribution of the queue length, orbit size and the inventory level and was calculated from the solution of the corresponding system of the linear balance equations. Markov QIS model studied in [8] has the finite queue of the p-customers where each customer after service completion either leaves the system or joins the orbit according to Bernoulli trial. The r-customers arriving from the orbit are taken to the service if there are no p-customers in the system and/or the inventory level is zero, while r-customers do not require the inventory item after service completion. The Q-matrix of the model was constructed and stationary distribution was calculated using the Matrix-Analyical methodology. The single server QIS model without queue was investigated in [9]. Arriving p-customers are taken to the service if the server is idle and inventory level is greater than zero. The customer leaves the system when there are no items left in the inventory and server is idle. If the server is busy the p-customer joins the orbit. The r-customers in the orbit are assumed to be insistent, so that if the server is busy or the inventory level is zero at the moment of arrival the r-customer returns back to the orbit. The corresponding three-dimensional Markov chain was constructed and solved using the matrix-geometric method.

The perishable QIS models (PQIS) with the repeated customers (feedback) were less investigated according to our literature research. The articles we found on the subject are [12] and [13]. The system with phase-type distributed positive service time and the MAP flow was studied in [13]. The inventory replenishment time (lead time), sojourn time in the orbit and the inventory perishing time are described with the exponentially distributed random variables (e.r.v). The system applies (s, S) replenishment policy and arriving customers are assumed to join the orbit the when waiting queue is full. The system is described using fivedimensional Markov chain. Algorithm based on the matrix-geometric method is developed to calculate the performance measures. In [12] considered the model with the simple Poisson flow, instantenous service time and server vacations. Likewise in [13], system applies the (s, S) for the inventory replenishment and the replenishment order lead time, orbit sojourn time and inventory perishing time are e.r.v. The server goes for a vacation whenever the inventory level reaches zero. The customers enter into the orbit of infinite size if the inventory level is zero or server is in a vacation at the arrival moment. The system was modeled using the three-dimensional Markov chain and solved using matrix-geometric method. In order to apply sojourn solution in [12] and [13] the r-customer arrival intensity are considered constant and that does not correspond to the reality.

PQIS model with feedback and instantenous service time was studied in [18] and detailed summary of PQIS models was provided in the article.

We consider the single server PQIS model with the positive service time, Poisson flow and infinite queue and orbit. The arriving p-customers are assumed to join the queue or the orbit or leave the system according to Bernoulli trial whenever the inventory level is zero. The waiting customers in the queue become impatient when the inventory reaches the zero level. The impatient customers leave the system or joins the orbit after some exponentially distributed random time. The arriving r-customers from the orbit either leaves the system or goes back to the orbit if there are no items left in the inventory. The served customers may or may not purchase the inventory item system after the service completion. The models with such customers were first considered in [7] and [14] and later applied in [15]. We use three-dimensional Markov chain to describe the mathematical model of the system. The stationary distribution and the performance measures of the system are calculated using the stochastic simulation algorithm [19].

The paper consists of five sections. In the first section the model description and the problem statement is provided. The mathematical model and the transition matrix (Q-matrix) of the system is given in the second section. The formulas for the calculation of the performance measures are developed as well. The problem solution using the Gillespie's Stochastic Simulation [19] algorithm is described in the third section and the application of other possible algorithms is analyzed. The umerical experiments are illustrated and discussed in the fourth section. The article is concluded with the summary of work done.

Model Description and Problem Statement

Let's consider the following M/M/1/∞/PQIS/RQ model. The identical p-customers arrive into the system according to Poisson scheme. The inventory level decreases by unit after service completion in case the customer purchases an item. The inventory items perish independently at random times described with e.r.v. The reserved item while servicing the customer does not perish and the customer does not leave the system while being served.

The arrived p-customer is taken to the service if the inventory level is positive and server is idle, otherwise the p-customer joins the queue of infinite size. If the inventory level is zero the arrived p-customer either joins the queue, enters the orbit or leaves the system according to Bernoulli trial. The r-customers in the orbit repeat their call after exponentially distributed random time.

During waiting in the queue customers become impatient when the inventory reaches zero level and the impatient customers leave the queue independently after the random time. After leaving the queue, the impatient customer either enters into the orbit or leaves the system according to Bernoulli trial. We assume that the cusomters may enter into the orbit multiple times.

The served customer may or may not purchase the inventory item after the service completion.

The system does not differentiate between the p-customers and r-customers, but the mean service time differs depending on whether the served customer purchases the inventory item or not (first-type and second-type customers).

System applies VSO policy. In order to be concretely, here up to S policy is used. When the inventory reaches the level s the replenishment order is placed. The lead-time is described with e.r.v. The inventory is replenished up to S level after the order delivery. We assume that $s < S/2$ in order to prevent multiple replenishment orders.

Problem statement is to find the joint stationary distribution of the inventory level, queue length and orbit size and to calculate the system performance measures.

The system has the following parameters:

λ – Poisson arrival intensity of the initial p-customers;

γ – inventory perishing intensity;

S – maximum inventory level;

s – replenishment order threshold, $s < S/2$;

v – replenishment order lead intensity;

ϕ_1 – the probability that arrived p-customer joins the queue if inventory level is zero;

ϕ_2 – the probability that arrived p-customer enters the orbit if inventory level is zero;

ϕ_3 – the probability that the p-customer leaves the system if inventory level is zero, $\phi_1 + \phi_2 + \phi_3 = 1$;

τ – queue leaving intensity of the impatient customers;

ψ_1 – the probability that the impatient customer leaves the system;

ψ_2 – the probability that the impatient customer enters into the orbit after leaving the queue, $\psi_1 + \psi_2 = 1$;

μ_1 – the service intensity of the first-type customer;

σ_1 – the probability that the customer does not purchase the item after service completion (first-type customer);

σ_2 – the probability that the customer purchases the item after service completion (second-type customer), $\sigma_1 + \sigma_2 = 1$;

μ_2 – the service intensity of the second-type customer;

η – the arrival intensity of r-customer from the orbit;

ξ_1 – the probability that the arrived r-customer leaves the system if inventory level is zero;

ξ_2 – the probability that the arrived r-customer goes back to the orbit if inventory level is zero, $\xi_1 + \xi_2 = 1$.

We divide the required performance measures into two related categories below:

Inventory related performance measures:

S_{av} – average inventory level;

RR – average replenishment rate;

R_{av} – average replenishment size;

Γ_{av} – average perishing rate;

Customer related performance measures:

L_s – average number of customers in queue;

L_o – average number of custemers in orbit;

RL_q– average customer loss intensity.

System is modeled using the continuous Markov chain and the current state is denoted as (m, n, k) where the variables m, n, k indicate the inventory level, the queue length and the orbit size correspondingly. The system State Space (*SS*) is defined as the follows:

$$E = \{(m,n,k) : m = 0,1, \ldots, S; n = 0,1\ldots; k = 0,1\ldots;\}. \quad (1)$$

We consider the following notations before constructing the transition rate matrix:

$p(m, n, k)$ – the stationary probability of the state (m, n, k);

$q(m_1, n_1, k_1),(m_2, n_2, k_2)$ – transition rate from the state (m_1, n_1, k_1) to the state (m_2, n_2, k_2).

The transition rate matrix of the system could be expressed with the parallelepiped of the finite base (inventory level), infinite height (orbit size) and depth (queue length). Let's group the transition rates for simplicity.

The first group consists of the transitions where the inventory level and queue length changes but the orbit size remains the same $(k_1 = k_2 = k)$. We differentiate two sub-groups here.

We put the transitions occurring from the state where the inventory level is positive $(m_1 > 0)$ into the first sub-group. Such transitions occur on the customer arrival, after the service completion, inventory perishing or replenishment delivery:

$$q((m_1,n_1,k), (m_2,n_2,k)) = \\ = \begin{cases} \lambda, & m_2 = m_1, n_2 = n_1 + 1; \\ \mu_1\sigma_1, & m_2 = m_1, n_2 = n_1 - 1; \\ \mu_2\sigma_2, & m_2 = m_1 - 1, n_2 = n_1 - 1; \\ m_1\gamma, & m_2 = m_1 - 1, \ n_2 = n_1 = 0; \\ (m_1 - 1)\gamma, & m_2 = m_1 - 1, n_2 = n_1 > 0; \\ v, & m_1 \le s, m_2 = S, n_2 = n_1; \\ O, & otherwise. \end{cases} \quad (2)$$

The second sub-group contains transitions occurring from the states where the inventory level is zero $(m_1 = 0)$. Such transitions occur on the customer arrival, after the impatient customer leaves the system and after the inventory replenishment:

$$q((m_1,n_1,k), (m_2,n_2,k)) = \\ = \begin{cases} \lambda\phi_2 & m_2 = 0, n_2 = n_1 + 1; \\ n_1\psi_1, & m_2 = 0, n_2 = n_1 - 1; \\ v, & m_2 = S, n_2 = n_1; \\ O, & otherwise. \end{cases} \quad (3)$$

The second group consists of the transitions where the orbit size changes $(k_1 \ne k_2)$ and the inventory level is not affected $(m_1 = m_2)$. Such transitions occur when the customers enter or leave the orbit:

$$q((m_1,n_1,k_1), (m_2,n_2,k_2)) = \\ = \begin{cases} \lambda\phi_2, & m_1 = 0, n_2 = n_1, k_2 = k_1 + 1; \\ n_1\tau\psi_2, & m_1 = 0, n_2 = n_1 - 1, k_2 = k_1 + 1; \\ k_1\eta, & m_1 > 0, n_2 = n_1 + 1, k_2 = k_1 - 1; \\ k_1\eta\xi_1, & m_1 = 0, n_2 = n_1, k_2 = k_1 - 1; \\ O, & otherwise. \end{cases} \quad (4)$$

We assume that $\lambda/\mu_1\sigma_1 < 1$ in order to ensure the ergodicity of the system. Then the stationary distribution exists and the following condition is hold:

$$\sum_{(m,n,k)\in E} p(m,n,k) = 1; \quad (5)$$

We may calculate the performance measure after the stationary probabilities are found. The inventory related performance measure are calculated with the following formulas:

$$S_{av} = \sum_{(m,n,k)\in E} mp(m,n,k); \quad (6)$$

$$\Gamma_{av} = \gamma\left(\sum_{m=1}^{S} m \sum_{(m,0,k)\in E} p(m,0,k) + \right. \\ \left. + \sum_{m=2}^{S}(m-1) \sum_{(m,n,k)\in E} p(m,n,k) I(n)\right); \quad (7)$$

$$RR = \gamma(s+1) \sum_{(s+1,0,k)\in E} p(s+1,0,k) + \\ + (\mu_2\sigma_2 + s\gamma) \sum_{(s+1,n,k)\in E} p(s+1,n,k) I(n); \quad (8)$$

$$R_{av} = \sum_{m=S-s}^{S} m \sum_{(n,k)\in E_m} p(S-m,n,k); \quad (9)$$

We derive the following formulas for the calculation of the customer related performance measures:

$$L_s = \sum_{(m,n,k)\in E} np(m,n,k); \quad (10)$$

$$L_O = \sum_{(m,n,k)\in E} kp(m,n,k); \quad (11)$$

$$RL_q = \tau\psi_1 \sum_{(0,n,k)\in E} np(0,n,k) + \\ + \eta\xi_1 \sum_{(0,n,k)\in E} kp(0,n,k) + \\ + \lambda\phi_3 \sum_{(0,n,k)\in E} p(0,n,k). \quad (12)$$

$I(A)$ is the indicator function of the event A, E_m is the set all the states where inventory level is m in the above formulas. (6), (9), (10), (11) are calculated as the mathematical expectation of the corresponding random variables. Other formulas are developed based on transition rate matrix of the system (see formulas (2)—(4)).

In conclusion, we introduced the mathematic model of the system, constructed the corresponding transition matrix and developed the formulas for the calculation of the performance measures. We will provide the algorithm to find the stationary distribution of the system in the next section.

Problem Solution

In order to find the stationary distribution of the Markov system we need to solve the system of linear balance equations. There are different algorithms to solve the infinite systems in special cases. For example, the algorithms like matrix-geometric method [10], spectral expansion [17] apply matrix algebra, eigenvalue and eigenvector calculations to solve the infinite system of linear equations under assumption of system ergodicity. These algorithms require computational power and their practical implementation and complexity may become problematic. Additionally, these methods imply some restictions on the transition matrix. Taking into account the form of our Q-matrix, these methods are not applicable in our case.

Let's consider the approximate SMA (Space Merging Algorithm) technology that is described in [18]. This method constructs the merged states based on each state parameter, divides the process into one-dimensional BD (Birth–Death) processes and reduces the dimensionality of the system. The stationary probabilities are expressed through the probabilities of the merged states based on the formula of conditional probability. This algorithm is computationally efficient, although it returns the approximate results. Additionally, it imposes some restrictions to the form of transition matrix. Our mathematical model is not appropriate for the application of this method.

When there is no analytical solution and the transition matrix is given, we may apply the well-known simulation algorithms to calculate the stationary distribution of the system. The examples of algorithms used for the simulation of the continuous Markov Chains are Gillespie's Direct method, Gibson Next Reaction method, Explicit Tau-Leap method. The more detailed information about simulation algorithms is provided in [20].

Let's apply the exact Gillespie's Direct [19] algorithm for the solution of our model. It consists of the following steps:

1. Define the overall simulation time T, set the initial time $t = 0$;
2. Define the initial state x of the system randomly or set it to zero state;
3. Calculate the sum of all the possible transition rates from the current state x:

$$Q = \sum_{x_i \in E} q(x, x_i), \; i = 0, \dots, l;$$

4. Draw the time duration Δt until the next event from the exponential distribution with parameters Q;
5. Generate random number r using the uniform distribution;
6. Set the next state x_n as follows:

   if $0 < r < q(x, x_0)/Q$, set $x = x_0$;

   if $q(x, x_0)/Q < r < \left(q(x, x_0) + q(x, x_1)\right)/Q$, set $x = x_1$;

   if $\sum_{i=0}^{n-1} q(x, x_i)/Q < r < \sum_{i=0}^{n} q(x, x_i)/Q$, set $x = x_n$;
7. Update the current time: $t = t + \Delta t$;

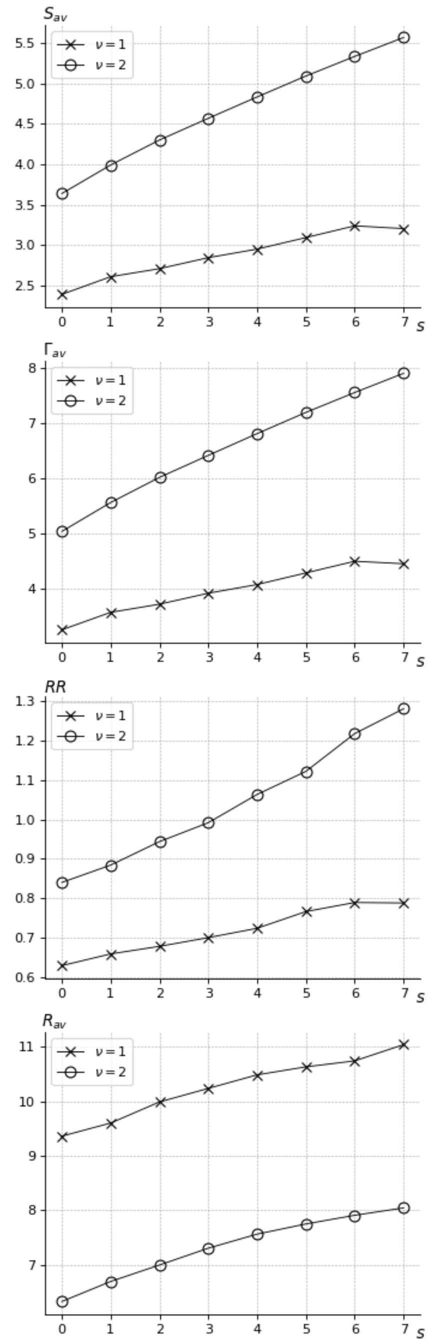

Fig. 1. Dependce graphs of inventory related performance measures (6)—(9)

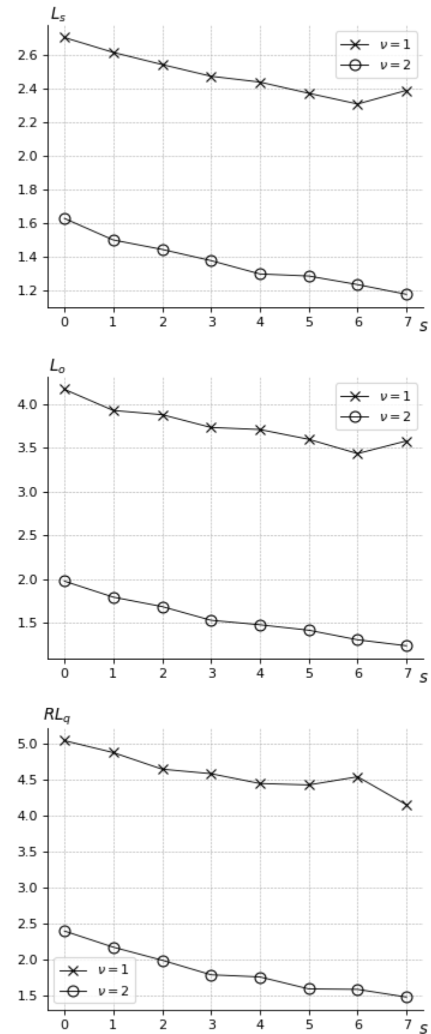

Fig. 2. Dependce graphs of customer related performance measures (10)—(12)

8. Repeat the steps 3—7 while $t \leq T$.

The simulation time T is choosen experimentally based on the system parameter values. The quantity Q in step 3 is calculated using the formulas (2)—(4). The stationary probability of each state is calculated as the ratio of its sojourn time to the total simulation time. After the stationary distribution is found we calculate the performance measures according to the formulas (6)—(12).

The simulation algorithms may become unefficient for the complex transition rate matrices. As concluded from the formulas (2)—(4) the transition matrix is appropriate for the simulation in our case.

In conclusion, we introduced the application of the simulation algorithm for the calculation of stationary distribution and the system performance measures in this section. We will analyze the numerical experiments in the next section.

Numerical Experiments

We will illustrate and discuss the numerical experiments obtained from the stochastic simulation of the system. We analyze the dependence of the system performance measures on the different replenishment lead times ($v = 1$, $v = 2$) and replenishment threshold.

The system parameters are assumed as following in the numerical experiments:
$\lambda = 15$; $\eta = 5$; $\mu_1 = 60$; $\mu_2 = 30$; $\sigma_1 = 0{,}3$; $\phi_1 = 0{,}4$; $\phi_2 = 0{,}5$; $\psi_1 = 0{,}2$; $\xi_1 = 0{,}2$; $\gamma = 1{,}5$; $\tau = 1{,}5$; $S = 15$.

We conclude from Fig. 1 that S_{av} increases linearly proportional to s. The reason is that inventory is replenished more frequently for the higher values of s which results in the increase of the average replenishment rate RR. Additionally, the intuitive relation $\Gamma_{av} \approx \gamma S_{av}$ holds true. The average replenishment amount R_{av}, in contrary to S_{av}, Γ_{av} and RR, decreases with the increase of the parameter v. Because with the higher lead intensity the average inventory level goes up that in turn lowers the overall replenishment amount.

We conclude from Fig. 2 that the average queue length and orbit size decreases with the increase of the critical level s. This explained with the higher S_{av} that reduces the probability of zero inventory level and allows serving the more customers. The lower probability of empty inventory results in the decrease of the customer loss intensity RL_q.

Conclusion

We have considered Perishable Queueing-Inventory system with positive service time and customer feedback. The queue length and orbit size are assumed infinite. The corresponding three-dimensional Markov chain and transition rate matrix are constructed. Stochastic simulation algorithm is applied to find the stationary distribution and calculate performance measures. Finally, the results of numerical results has been illustrated and discussed.

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Мамед Шахмалиев, аспирант,
Национальная академия авиации,
ул. Худу Мамедова, 38, 122, Баку, AZ1129, Азербайджан,
mamed.shahmaliyev@gmail.com

ИМИТАЦИОННАЯ МОДЕЛЬ СИСТЕМЫ ОБСЛУЖИВАНИЯ–ЗАПАСАНИЯ С БЕСКОНЕЧНОЙ ОЧЕРЕДЬЮ И ОБРАТНОЙ СВЯЗЬЮ

Введение. Рассмотрена система обслуживания-запасания с бесконечной очередью, положительным временем обслуживания и отзывами клиентов. Размер заказов конечен, а длина очереди и орбит бесконечна. Клиенты, прибывающие в соответствии с потоками Пуассона, либо покупают товарный запас либо покидают систему. Клиенты в очереди становятся нетерпеливыми, когда уровень запасов падает до нуля. Нетерпеливые клиенты в соответствии с испытанием Бернулли оставляют систему присоединения к орбите после экспоненциально распределенного случайного времени. Если в инвентаре нет предметов в момент прибытия, клиент либо входит в очередь или на орбиту, либо покидает систему в соответствии с распределением Бернулли. В системе применяется политика порядка сортировки для пополнения запасов.

Методы. Использован метод стохастического моделирования для расчета показателей производительности системы и поиска ее стационарного распределения. Зависимость показателей эффективности от уровня переупорядочения проиллюстрирована и проанализирована с использованием численных экспериментов.

Результат. Предложена имитационная модель системы обслуживания-запасания с положительным временем обслуживания, бесконечной очередью, бесконечной орбитой и обратной связью. В системе обслуживаются заявки двух типов и используется стратегия пополнения запасов с переменным объемом заказов. Время выполнения заказов — случайная величина с показательной функцией распределения. Разработана соответствующая трехмерная цепь Маркова и предложен алгоритм построения ее производящей матрицы. Получены формулы для расчета характеристик системы. Построена имитационная модель для поиска стационарного распределения данной цепи Маркова.

Выводы. Путем численных экспериментов изучены зависимости характеристик системы от критического уровня запасов и времени выполнения заказов.

***Ключевые слова**: система обслуживания-запасания, бесконечная трехмерная Марковская сеть, алгоритм моделирования, VSO-политика, численные эксперименты.*

Мамед Шахмалієв, аспірант,
Національна академія авіації,
вул. Худу Мамедова, 38, 122, Баку, AZ1129, Азербайджан,
mamed.shahmaliyev@gmail.com

ІМІТАЦІЙНА МОДЕЛЬ СИСТЕМИ ОБСЛУГОВУВАННЯ-ЗАПАСАННЯ
З НЕСКІНЧЕННОЮ ЧЕРГОЮ ТА ЗВОРОТНИМ ЗВ'ЯЗКОМ

Вступ. Розглянуто систему обслуговування-запасання з нескінченною чергою, позитивним часом обслуговування та відгуками клієнтів. Розмір замовлень кінцевий, а довжина черги і орбіт нескінченна. Клієнти, які прибувають відповідно до потоків Пуассона, або купують товарний запас, або залишають систему. Клієнти в черзі стають нетерплячими, коли рівень запасів падає до нуля. Нетерплячі клієнти відповідно до випробування Бернуллі залишають систему приєднання до орбіти після експоненціально розподіленого випадкового часу. Якщо в інвентарі немає предметів в момент прибуття, клієнт або входить в чергу або на орбіту, або залишає систему відповідно до розподілу Бернуллі. У системі застосовується політика порядку сортування для поповнення запасів.

Методи. Використано метод стохастичного моделювання для розрахунку показників продуктивності системи і пошуку її стаціонарного розподілу. Залежність показників ефективності від рівня переупорядковування проілюстровано і проаналізовано з використанням чисельних експериментів.

Результат. Запропоновано імітаційну модель системи обслуговування-запасання з позитивним часом обслуговування, нескінченною чергою, нескінченною орбітою і зворотним зв'язком. В системі обслуговуються заявки двох типів і використовується стратегія поповнення запасів із змінним обсягом замовлень. Час виконання замовлень є випадковою величиною з показовою функцією розподілу. Запропоновано імітаційну модель системи обслуговування-запасання з позитивним часом обслуговування, нескінченною чергою, нескінченною орбітою і зворотним зв'язком. В системі обслуговуються заявки двох типів і використовується стратегія поповнення запасів із змінним обсягом замовлень. Час виконання замовлень є випадковою величиною з показовою функцією розподілу. Розроблено відповідний тривимірний ланцюг Маркова і запропоновано алгоритм побудови її відтворювальної матриці. Отримано формули для розрахунку характеристик системи. Побудовано імітаційну модель для пошуку стаціонарного розподілу даного ланцюга Маркова.

Висновок. Шляхом чисельних експериментів вивчено залежності характеристик системи від критичного рівня запасів і від часу виконання замовлень.

***Ключові слова**: система обслуговування-запасання, нескінченна тривимірна Марковська мережа, алгоритм моделювання, VSO-політика, чисельні експерименти.*