UDC 519.718

B.Ye. Rytsar

A Simple Minimization Method of the Variables Number in Complete and Incomplete Logic Functions. Part 1

Предложен новый метод минимизации числа переменных в полных и неполных логических функциях, основанный на процедуре расцепления конъюнктермов. Преимущества предложенного метода иллюстрируют примеры определения несущественных переменных в функциях, заимствованных автором из известных публикаций с целью сравнения.

Ключевые слова: минимизация числа переменных, логическая функция, несущественная переменная, конъюнктерм, процедура расцепления.

Запропоновано новий метод мінімізації кількості змінних у повних і неповних логічних функціях, що ґрунтується на процедурі розчеплення кон'юнктермів. Переваги пропонованого методу ілюструють приклади визначення неістотних змінних у функціях, які автор запозичив з відомих публікацій з метою порівняння.

Ключові слова: мінімізації кількості змінних, логічна функція, неістотна змінна, кон'юнктерм, процедура розчеплення.

A new minimization method of the variables number in complete and incomplete logic functions, based on the procedure of conjuncterms splitting is proposed. The advantages of the proposed method are illustrated by examples of determining nonessential variables in the functions, which are borrowed from the well-known publications.

Keywords: minimization of the variables number, logic function, nonessential variable, conjuncterm, splitting procedure.

Introduction. Detection and reduction of variables that for the logic function are not essential as they do not change its value after their elimination is an important pre-procedure process of logic synthesis of digital devices and systems [1–9]. The reduction of nonessential variables in incomplete (incompletely specified) functions is called minimization of the number of variables that is essentially different from the minimization of the function itself [2, 3]. Such reduction in a given function is important especially for the problems of the function minimization as evaluation of the complexity (the cost) of the synthesized device implementation depends on the number of variables.

The problem of minimization of the variables number in complete and incomplete functions by eliminating nonessential variables was analyzed in [1, 3]. The procedure for the reduction of nonessential variables is much more complicated for incomplete, especially weakly determinated, functions [1, 3–6]. The known tabular methods (including K-maps) [3–7], analytical methods based on the Shannon expansion [3, 6, 8], heuristic methods [1, 4, 8], methods based on the functional decomposition [7, 8], on decomposition clones [9] etc. are very complex in their practical implementation. Thus, the tabular and analytical methods have an obvious disadvantage related to their implementation (the dimension of given function). On the contrary, the heuristic methods are cumbersome (heavy) to implement since in order to eliminate a variable that at first is considered as nonessential, for each minterm of function there is an artificially introduced adjacent minterm to detect if the function is preserved or destroyed. However, as noted in [1], the reliability of the result is not guaranteed, due to its dependence of the removal sequence (relatively nonessential) of variables. The mentioned disadvantages of the known methods are particularly notable for weakly determinated functions and their systems.

This paper proposes a new method for the reduction of the number of variables in complete and incomplete functions and their systems. It is based on the method of conjuncterms splitting [10, 11] but differs in the fact that for the detection of nonessential variables a relatively simple procedure is used for both complete and incomplete (in predeterminated and weakly determinated) functions and their systems. The presented approach provides simultaneous execution of the predetermination procedure for incomplete functions allowing the simplification for the search of nonessential variables as for one function as well as for a system of functions.

1. **The theoretical basis of the method**. The conjuncterms splitting method of r-rank $\theta_1^r, \theta_2^r, ..., \theta_k^r$ of the logic function $f(x_1, x_2, ..., x_n)$, where $\theta_i^r = (\sigma_1\sigma_2 \cdots \sigma_n)$, $\sigma_j \in \{0,1,-\}$, $r \in \{2,3,...,n\}$, is based on the idea of sequential replacing of all their binary values by one, two, ..., $(n-1)$ dashes (–) setting the masks of literals $\{ll \cdots l\}$ of the ranks lower than the initial rank r [10,11]. In particular, when to impose all the masks of literals of $(n-1)$–, $(n-2)$–, …, r-ranks on a minterm (i.e. conjuncterm of n-rank) of a function f, it generates the set of conjuncterms splitting of $(n-1)$–, $(n-2)$–, …, r-ranks. This is illustrated by the following example of the binary minterm (1001) of a function $f(x_1, x_2, x_3, x_4)$:

$$(1001) \overset{S}{\Rightarrow} \begin{Bmatrix} lll- \\ ll-l \\ l-ll \\ -lll \end{Bmatrix} = \begin{bmatrix} 100- \\ 10-1 \\ 1-01 \\ -001 \end{bmatrix},\ (1001) \overset{S}{\Rightarrow} \begin{Bmatrix} ll-- \\ l-l- \\ l--l \\ -ll- \\ -l-l \\ --ll \end{Bmatrix} = \begin{bmatrix} 10-- \\ 1-0- \\ 1--1 \\ -00- \\ -0-1 \\ --01 \end{bmatrix},\ (1001) \overset{S}{\Rightarrow} \begin{Bmatrix} l--- \\ -l-- \\ --l- \\ ---l \end{Bmatrix} = \begin{bmatrix} 1--- \\ -0-- \\ --0- \\ ---1 \end{bmatrix},$$

where $\overset{S}{\Rightarrow}$ is a symbol of splitting procedure.

Accordingly, if conjuncterm is of r-rank, for example of 3-rank $(1-01)$, we obtain

$$(1-01) \overset{S}{\Rightarrow} \begin{Bmatrix} l-l- \\ l--l \\ --ll \end{Bmatrix} = \begin{bmatrix} 1-0- \\ 1--1 \\ --01 \end{bmatrix},\ (1-01) \overset{S}{\Rightarrow} \begin{Bmatrix} l--- \\ --l- \\ ---l \end{Bmatrix} = \begin{bmatrix} 1--- \\ --0- \\ ---1 \end{bmatrix}.$$

Let the function $f(x_1, x_2, ..., x_n)$ is given in the set-theoretical form (STF) of a set of minterms $m_1, m_2, ..., m_k$ as a perfect STF $Y^1 = \{m_1, m_2, ..., m_k\}^1$, $2 \le k < 2^n$ [10,11]. After imposition of all masks of $\binom{n}{n-1}$ literals of $(n-1)$-rank on the k minterms a splitting matrix is derived

$$M_n^{n-1} = \left[\theta_{ij}^{n-1}\right]_{n\times k}, \tag{1}$$

where matrix elements will be $n \times k$ splitted conjuncterms of $(n-1)$-rank θ_{ij}^{n-1}.

In the matrix (1) the identical pairs of elements can be formed that are conjuncterms-copies of $(n-1)$-rank and we will further underline them. They define a minimal coverage of the matrix (1), necessary to obtain the desired result. The set of matrix covering is final when its elements do not form conjuncterms-copies during the next step of the splitting procedure. Note, that the splitting procedure of conjuncterms underlies the known minimization method of logic functions. This algorithm is implemented in SPLIT v.2.3 software [10].

The splitting procedure is illustrated by an example of a function $f(x_1, x_2, x_3)$ given perfect STF $Y^1 = \{(001),(010),(011),(100)\}^1$. After imposition of all masks of literals of $(n-1)$-rank on the given minterms, we obtain a matrix M_2^3, where the allocated conjuncterms-copies of 2-rank are underlined:

$$Y^1 = \{(001),(010),(011),(100)\}^1 \overset{S}{\Rightarrow} \begin{Bmatrix} ll- \\ l-l \\ -ll \end{Bmatrix} = \begin{bmatrix} 00- & \underline{01-} & \underline{01-} & 10- \\ \underline{0-1} & 0-0 & \underline{0-1} & 1-0 \\ -01 & -10 & -11 & -00 \end{bmatrix} \overset{C}{\Rightarrow}$$

$$\overset{C}{\Rightarrow} \{(01-),(0-1),(100)\}^1,$$

where $\overset{C}{\Rightarrow}$ is a coverage operator of matrix splitting M_2^3.

Nonessential variables of the function f can be easily identified by the splitting procedure for the given minterms, starting with the matrix M_n^{n-1}.

2. **Definition of nonessential variables in the complete functions**. Let us formulate the theorem for the case of complete function $f:\{0,1\}^n \to \{0,1\}$.

Theorem. A logic function $f(x_1, x_2, ..., x_n)$ is given by a perfect STF $Y^1 = \{m_1, m_2, ..., m_k\}^1$, $m_i = (\sigma_1 \cdots \sigma_p \cdots \sigma_n)_i$, $\sigma_j \in \{0,1\}$, here $2 \le k < 2^{n-1}$ is an even number, has a nonessential variable x_p if and only if the imposition of all masks of literals of $(n-1)$-rank on the *k* given minterms $m_1, m_2, ..., m_k$ is done, the splitting matrix M_n^{n-1} is formed and is completely covered by the splitting conjuncterms-copies of $(n-1)$-rank θ_{i1}^{n-1}, θ_{i2}^{n-1},…, θ_{ik}^{n-1} of *i*-th row created by the mask of literals $\{l_1 \cdots l_{p-1} \cdots (-)_p \cdots l_{p+1} \cdots l_n\}_i$.

Proof. Evidence of this assertion is based on the fact that any one-pair of conjuncterms-copies of $(n-1)$-rank in splitting matrix M_n^{n-1} is formed by two adjacent (generating) minterms that differ in the value of the same binary position [10]. The position of the dash (–) in these elements of the matrix M_n^{n-1} is specified by certain mask of literals of $(n-1)$-rank. Therefore, if the matrix M_n^{n-1} has a *i*-th row of conjuncterms-copies of $(n-1)$-rank $\theta_{i1}^{n-1}, \theta_{i2}^{n-1}, \ldots, \theta_{ik}^{n-1}$, created by the mask $\{l_1 \cdots l_{p-1} \cdots (-)_p \cdots l_{p+1} \cdots l_n\}_i$ that completely cover the matrix (and this is only possible for even *k*), then the derived set covering the matrix M_n^{n-1} will have a dash in the same binary position *p* that will finally define a nonessential variable x_p of a function *f*. In the case of odd *k* of minterms all variables of complete function *f* are essential. □

The following example illustrates the validity of this Theorem.

Example 1. To determine nonessential variables using the splitting method in the complete function $f(x_1, x_2, x_3, x_4, x_5)$ given in the perfect STF $Y^1 = \{2, 3, 6, 7, 8, 10, 18, 19, 22, 23, 24, 26\}^1$ (*this function is borrowed from* [1, p. 270])

Solution. As a result of the imposition of all (five) masks of 4-rank literals on binary minterms of the given function *f* the splitting matrix M_5^4 formed:

$$Y^1 = \{(00010),(00011),(00110),(00111),(01000),(01010),(10010),(10011),(10110),(10111),(11000),(11010)\}^1 \overset{S}{\Rightarrow}$$

$$\begin{Bmatrix} IIII- \\ III-I \\ II-II \\ I-III \\ -IIII \end{Bmatrix} = \begin{bmatrix} \underline{0001-} & \underline{0001-} & \underline{0011-} & \underline{0011-} & 0100- & 0101- & \underline{1001-} & \underline{1001-} & \underline{1011-} & \underline{1011-} & 1100- & 1101- \\ 000-0 & 000-1 & 001-0 & 001-1 & \underline{010-0} & \underline{010-0} & 100-0 & 100-1 & 101-0 & 101-1 & \underline{110-0} & \underline{110-0} \\ \underline{00-10} & \underline{00-11} & \underline{00-10} & \underline{00-11} & 01-00 & 01-10 & \underline{10-10} & \underline{10-11} & \underline{10-10} & \underline{10-11} & 11-00 & 11-10 \\ \underline{0-010} & 0-011 & 0-110 & 0-111 & 0-000 & \underline{0-010} & \underline{1-010} & 1-011 & 1-110 & 1-111 & 1-000 & \underline{1-010} \\ \underline{-0010} & \underline{-0011} & \underline{-0110} & \underline{-0111} & \underline{-1000} & \underline{-1010} & \underline{-0010} & \underline{-0011} & \underline{-0110} & \underline{-0111} & \underline{-1000} & \underline{-1010} \end{bmatrix}.$$

Since the matrix M_5^4 is completely covered by the last row for the mask of literals $\{-IIII\}$, namely:

$$\overset{C}{\Rightarrow} \{(-0010),(-0011),(-0110),(-0111),(-1000),(-1010)\}^1,$$

then, according to the Theorem, the given function *f* has the nonessential variable x_1.

If necessary, the minimal STF Y^1 function $f(-, x_2, x_3, x_4, x_5)$ can be obtained as follows:

$$Y^1 = \{(-0010),(-0011),(-0110),(-0111),(-1000),(-1010)\}^1 \overset{S}{\Rightarrow}$$

$$\overset{S}{\Rightarrow} \begin{Bmatrix} -III- \\ -II-I \\ -I-II \\ --III \end{Bmatrix} = \begin{bmatrix} \underline{-001-} & \underline{-001-} & \underline{-011-} & \underline{-011-} & -100- & -101- \\ -00-0 & -00-1 & -01-0 & -01-1 & \underline{-10-0} & \underline{-10-0} \\ \underline{-0-10} & \underline{-0-11} & \underline{-0-10} & \underline{-0-11} & -1-00 & -1-10 \\ \underline{--010} & --011 & --110 & --111 & --000 & \underline{--010} \end{bmatrix} \overset{C}{\Rightarrow}$$

$$\overset{C}{\Rightarrow} \{(-001-),(-011-),(-10-0)\}^1 \Rightarrow \{(-0-1-),(-10-0)\}^1.$$

Answer. The given function $f(-,x_2,x_3,x_4,x_5)=\bar{x}_2x_4 \vee x_2\bar{x}_3\bar{x}_5$, that corresponds to [1].

Corollary of Theorem. If the splitting matrix M_n^{n-1} of the given k minterms $m_1,m_2,...,m_k$ of the perfect STF Y^1 of the function f has two, three, ...,$(n-1)$ rows of the conjuncterms-copies of $(n-1)$-rank, each of which covers it completely, then this function f has respectively two, three, …,$(n-1)$ nonessential variables; in the case of n rows we have degenerated the function f.

This case is illustrated below by an example of the function $f(x_1,x_2,x_3,x_4,x_5)$ given in perfect STF $Y^1=\{2,3,6,7,24,25,28,29\}^1$. Splitting of the given minterms by the matrix M_5^4, we obtain

$$Y^1=\{(00010),(00011),(00110),(00111),(11000),(11001),(11100),(11101)\}^1 \overset{S}{\Rightarrow}$$

$$\overset{S}{\Rightarrow}\begin{Bmatrix} llll- \\ lll-l \\ ll-ll \\ l-lll \\ -llll \end{Bmatrix}=\begin{bmatrix} \underline{0001-} & \underline{0001-} & \underline{0011-} & \underline{0011-} & \underline{1100-} & \underline{1100-} & \underline{1110-} & \underline{1110-} \\ 000-0 & 000-1 & 001-0 & 001-1 & 110-0 & 110-1 & 111-0 & 111-1 \\ \underline{00-10} & \underline{00-11} & \underline{00-10} & \underline{00-11} & \underline{11-00} & \underline{11-01} & \underline{11-00} & \underline{11-01} \\ 0-010 & 0-011 & 0-110 & 0-111 & 1-000 & 1-001 & 1-100 & 1-101 \\ -0010 & -0011 & -0110 & -0111 & -1000 & -1001 & -1100 & -1101 \end{bmatrix}\overset{C}{\Rightarrow}$$

$$\overset{C}{\Rightarrow}\begin{cases} \{(0001-),(0011-),(1100-),(1110-)\}^1 \\ \{(00-10),(00-11),(11-00),(11-01)\}^1 \end{cases}.$$

Since the splitting matrix M_5^4 has two rows for the masks $\{llll-\}$ and $\{ll-ll\}$, each of which covers it completely, then according to the corollary of the Theorem the given function f has the two nonessential variables x_5 and x_3. Performing the splitting procedure on the elements of the obtained sets by the matrix M_5^3, we will get the minimal STF Y^1 of the function f:

$$\{(0001-),(0011-),(1100-),(1000-)\}^1 \overset{S}{\Rightarrow}\begin{Bmatrix} lll-- \\ ll-l- \\ l-ll- \\ -lll- \end{Bmatrix}=\begin{bmatrix} 000-- & 001-- & 110-- & 111-- \\ \underline{00-1-} & \underline{00-1-} & \underline{11-0-} & \underline{11-0-} \\ 0-01- & 0-11- & 1-00- & 1-10- \\ -001- & -011- & -100- & -110- \end{bmatrix}\overset{C}{\Rightarrow}$$

$$\{(00-10),(00-11),(11-00),(11-01)\}^1 \overset{S}{\Rightarrow}\begin{Bmatrix} ll-l- \\ ll--l \\ l--ll \\ -l-ll \end{Bmatrix}=\begin{bmatrix} \underline{00-1-} & \underline{00-1-} & \underline{11-0-} & \underline{11-0-} \\ 00--0 & 00--1 & 11--0 & 11--1 \\ 0--10 & 0--11 & 1--00 & 1--01 \\ -0-10 & -0-11 & -1-00 & -1-01 \end{bmatrix}\overset{C}{\Rightarrow}$$

$$\overset{C}{\Rightarrow}\{(00-1-),(11-0-)\}^1.$$

Answer. Minimal STF $Y^1=\{(00-1-),(11-0-)\}^1 \equiv \{(2,3,6,7),(24,25,28,29)\}^1$, that corresponds to $f(x_1,x_2,-,x_4,-)=\bar{x}_1\bar{x}_2x_4 \vee x_1x_2\bar{x}_4$.

3. **Definition of nonessential variables in the incomplete functions**. The definition of nonessential variables in the incomplete functions (unlike the complete functions) belongs to the complex multivariate optimization problems [1, 3]. The solution of the problem by the proposed method is also based on the Theorem and its corollary but taking into account peculiarities of the incomplete functions.

The incomplete function $f:\{0,1\}^n \to \{0,1,\sim\}$, where sign (tilde) ~ symbolizes the «don't care», i.e. «undefined» value of the function f, in the set-theoretic format is represented by two sets that make up the perfect STF

$$\begin{cases} Y^1=\{m_1,m_2,...,m_r\}^1 \\ Y^{\sim}=\{m_{r+1},m_{r+2},...,m_{2^n-r-h}\}^{\sim} \end{cases} \text{ or } \begin{cases} Y^1=\{m_1,m_2,...,m_r\}^1 \\ Y^0=\{m_{r+1},m_{r+2},...,m_{2^n-r-h}\}^0 \end{cases}, \quad h<2^n-r, \qquad (2),(3)$$

Where Y^1, Y^0 and $Y^{\sim}$ are subsets of minterms of the full set $\mathbf{E}_2^n$, on which the function *f* takes the value respectively 1, 0 and «don't care» (~) [10]. If $|Y^1 \cup Y^0| \geq |Y^{\sim}|$, then such the inpredeterminated (incomplete) function *f* given by minterms of perfect STF $\{Y^1, Y^{\sim}\}$ (2), and if $|Y^1 \cup Y^0| < |Y^{\sim}|$, then the weakly determinated (incomplete) function *f* given by minterms of perfect STF $\{Y^1, Y^0\}$ (3). Accordingly, the splitting matrix M_n^r, $r \in \{1,2,...,n-1\}$, will consists of two submatrices – basic (for Y^1) and additional (for $Y^{\sim}$ or Y^0) that will be separatedby the symbol $\vdots$. Thus, if the given function *f* is inpredeterminated we have $Y^1 \vdots Y^{\sim}$, and if the given function *f* is weakly determinated we have $Y^1 \vdots Y^0$.

The algorithm of definition of the nonessential variables in the incomplete functions *f* will be implemented as follows. The conjuncterms-copies in the basic submatrix (for Y^1) are under lined as well as in the case of the complete function. Therefore, the double underlining is applied in the case of an inpredeterminated function *f* to the conjuncterms-copies which (one by one) belong to two submatrices Y^1 and $Y^{\sim}$, and in the case of the weakly determinated function *f* to the elements of the submatrix Y^1 that do not have copies of the submatrix Y^0. In such a way, the procedure of predetermination of the incomplete functions is implemented. While in both cases for a certain mask of literals all elements in the submatrix Y^1 are allocated, then according to the Theorem we obtain the desired result: a dash (–) in the same position of elements covering the submatrix Y^1 indicates the nonessential variable in the incomplete function *f*.

Definition of nonessential variables in the incomplete functions by the proposed method is illustrated by the following example of the function $f(x_1, x_2, x_3)$.

Let the inpredeterminated function *f* be given in the perfect STF $\begin{cases} Y^1 = \{1,3,4,5\}^1 \\ Y^{\sim} = \{6,7\}^{\sim} \end{cases}$ and the weakly determinated function *f* be given in the perfect STF $\begin{cases} Y^1 = \{1,3,4,5\}^1 \\ Y^0 = \{0,2\}^0 \end{cases}$. In both cases, the splitting procedure of the given minterms is performed with the matrix M_3^2.

For the inpredeterminated function *f* we have:

$$Y^1 \vdots Y^{\sim} = \{(001),(011),(100),(101)\}^1 \vdots \{(110),(111)\}^{\sim} \overset{S}{\Rightarrow}$$

$$\overset{S}{\Rightarrow} \begin{Bmatrix} ll- \\ l-l \\ -ll \end{Bmatrix} = \left[\begin{array}{cccc:cc} 00- & 01- & \underline{10-} & \underline{10-} & 11- & 11- \\ \underline{0-1} & \underline{0-1} & \underline{\underline{1-0}} & \underline{\underline{1-1}} & \underline{\underline{1-0}} & \underline{\underline{1-1}} \\ \underline{-01} & \underline{\underline{-11}} & -00 & \underline{-01} & -10 & \underline{\underline{-11}} \end{array} \right] \overset{C}{\Rightarrow} \{(0-1),(1-0),(1-1)\}^1 \Rightarrow \{(--1),(1--)\}^1,$$

where according to the Theorem the nonessential variable is x_2, since the matrix M_3^2 is covered by the splitted conjuncterms of the 2-rank of the submatrix for Y^1 which are formed by the mask $\{l-l\}$.

In the case of the weakly determinated function *f* we will have the same result:

$$Y^1 \vdots Y^0 = \{(001),(011),(100),(101)\}^1 \vdots \{(000),(010)\}^0 \overset{S}{\Rightarrow}$$

$$\overset{S}{\Rightarrow} \begin{Bmatrix} ll- \\ l-l \\ -ll \end{Bmatrix} = \left[\begin{array}{cccc:cc} 00- & 01- & \underline{10-} & \underline{10-} & 00- & 01- \\ \underline{0-1} & \underline{0-1} & \underline{\underline{1-0}} & \underline{\underline{1-1}} & 0-0 & 0-0 \\ \underline{-01} & \underline{\underline{-11}} & -00 & \underline{-01} & -00 & -10 \end{array} \right] \overset{C}{\Rightarrow} \{(0-1),(1-0),(1-1)\}^1 \Rightarrow \{(--1),(1--)\}^1.$$

Therefore, the given function *f* has the nonessential variable x_2, i.e. $f(x_1, -, x_3) = x_1 \vee x_3$.

Example 2. To determine nonessential variables with the splitting method in the incomplete function $f(x_1, x_2, x_3, x_4, x_5)$ given in the analytical expressions:

$$F_1 = \bar{x}_5\{\bar{x}_1\bar{x}_2x_3 \vee x_1\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_4\} \vee x_5\{x_1\bar{x}_2\bar{x}_3 \vee x_1\bar{x}_2\bar{x}_4 \vee \vee x_1x_2x_3x_4 \vee \bar{x}_1\bar{x}_2x_3x_4\}$$

$$F_0 = \bar{x}_5\{\bar{x}_1x_2(x_3 \vee \bar{x}_4) \vee x_1x_2(x_3 \oplus x_4)\} \vee x_5\{x_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\}$$

(*this function is borrowed from* [3, *example* 11.1, p.133]).

Solution. The given incomplete function *f* is represented by the perfect STF

$$\begin{cases} Y^1 = \{(00100),(00110),(00111),(10000),(10001),(10010),(10011),(10100),(10101),(11000),(11111)\}^1 \\ Y^0 = \{(00001),(01000),(01001),(01011),(01100),(01011),(01110),(11010),(11011),(11100)\}^1 \end{cases}$$

Let us perform the splitting procedure of the given minterms using the matrix M_5^4. The conjuncterms-copies in the submatrix Y^1 are underlined and the double underlining is for the conjuncterms with no copies in the submatrix Y^0 (i.e. their copies are in the submatrix $Y^{\sim}$):

$$Y^1 \vdots Y^0 \overset{S}{\Rightarrow}$$

$$\begin{Bmatrix} llll- \\ lll-l \\ ll-ll \\ l-lll \\ -llll \end{Bmatrix} = \left[\begin{array}{ccccccccccc|} \underline{\underline{0010-}} & \underline{0011-} & \underline{0011-} & \underline{1000-} & \underline{1000-} & \underline{1001-} & \underline{1001-} & \underline{1010-} & \underline{1010-} & \underline{\underline{1100-}} & \underline{\underline{1111-}} \\ \underline{001-0} & \underline{001-0} & \underline{\underline{001-1}} & \underline{100-0} & \underline{100-1} & \underline{100-0} & \underline{100-1} & \underline{\underline{101-0}} & \underline{\underline{101-1}} & 110-0 & \underline{\underline{111-1}} \\ \underline{\underline{00-00}} & \underline{\underline{00-10}} & \underline{\underline{00-11}} & \underline{10-00} & \underline{10-01} & \underline{\underline{10-10}} & \underline{\underline{10-11}} & \underline{10-00} & \underline{\underline{10-01}} & 11-00 & \underline{\underline{11-11}} \\ 0-100 & 0-110 & \underline{\underline{0-111}} & \underline{1-000} & \underline{\underline{1-001}} & 1-010 & 1-011 & 1-100 & \underline{\underline{1-101}} & \underline{1-000} & \underline{\underline{1-111}} \\ \underline{-0100} & \underline{\underline{-0110}} & \underline{\underline{-0111}} & \underline{\underline{-0000}} & -0001 & \underline{\underline{-0010}} & \underline{\underline{-0011}} & \underline{-0100} & \underline{\underline{-0101}} & -1000 & \underline{\underline{-1111}} \end{array}\right.$$

$$\left.\begin{array}{|cccccccccc} 0000- & 0100- & 0100- & 0101- & 0110- & 0110- & 0111- & 1101- & 1101- & 1110- \\ 000-1 & 010-0 & 010-1 & 010-1 & 011-0 & 011-1 & 011-0 & 110-0 & 110-1 & 111-0 \\ 00-01 & 01-00 & 01-01 & 01-11 & 01-00 & 01-01 & 01-10 & 11-10 & 11-11 & 11-00 \\ 0-001 & 0-000 & 0-001 & 0-011 & 0-100 & 0-101 & 0-110 & 1-010 & 1-011 & 1-100 \\ -0001 & -1000 & -1001 & -1011 & -1100 & -1101 & -1110 & -1010 & -1011 & -1100 \end{array}\right].$$

As one can see, the submatrix Y^1 has only one row for the mask $\{llll-\}$ covered by selected elements. This indicates that, according to the Theorem, the given function *f* has the nonessential variable x_5, that corresponds to [3].

As it was noted above, between the minimization of a function and the minimization of its number of variables there is a fundamental difference. Both of these concepts are easily distinguished by the proposed method. We demonstrate it by an example of the weakly determinated function $f(x_1, x_2, x_3, x_4)$ given in the perfect STF $\begin{cases} Y^1 = \{5,9,12\}^1 \\ Y^0 = \{1,6,8\}^0 \end{cases}$ (*this function is borrowed from* [3, p. 120 and 7, p. 45].

As a result of splitting of the given minterms we obtain:

$$Y^1 \vdots Y^0 = \{(0101),(1001),(1100)\}^1 \vdots \{(0001),(0110),(1000)\}^0 \overset{S}{\Rightarrow}$$

$$\overset{S}{\Rightarrow} \begin{Bmatrix} lll- \\ ll-l \\ l-ll \\ -lll \end{Bmatrix} = \left[\begin{array}{ccc|ccc} \underline{\underline{010-}} & 100- & \underline{\underline{110-}} & 000- & 011- & 100- \\ \underline{\underline{01-1}} & \underline{\underline{10-1}} & \underline{\underline{11-0}} & 00-1 & 01-0 & 10-0 \\ 0-01 & \underline{\underline{1-01}} & 1-00 & 0-01 & 0-10 & 1-00 \\ \underline{\underline{-101}} & -001 & \underline{\underline{-100}} & -001 & -110 & -000 \end{array}\right] \overset{C}{\Rightarrow} \{(01-1),(10-1),(11-0)\}^1.$$

As the submatrix Y^1 of the splitting matrix M_4^3 is covered by conjuncterms-copies, formed by the mask $\{ll-l\}$, then according to the Theorem the given function *f* has the nonessential variable x_3.

When continuing the splitting procedure of the obtained conjuncterms by the matrix M_4^2 we obtain the minimal STF Y^1 of the given function *f* with the nonessential variable x_3:

$$\overset{S}{\Rightarrow}\begin{bmatrix} ll-- \\ l--l \\ -l-l \end{bmatrix} = \left[\begin{array}{ccc:ccc} 01-- & 10-- & \underline{\underline{11--}} & 00-- & 01-- & 10-- \\ 0--1 & \underline{\underline{1--1}} & 1--0 & 0--1 & 0--0 & 1--0 \\ \underline{\underline{-1-1}} & -0-1 & -1-0 & -0-1 & -1-0 & -0-0 \end{array}\right] \overset{C}{\Rightarrow} \{(-1-1),(1--1),(11--)\}^1 \Rightarrow$$

$$\Rightarrow \{(5,\mathbf{7},\mathbf{13},\mathbf{15}),(9,\mathbf{11},\mathbf{13},\mathbf{15}),(12,\mathbf{13},\mathbf{14},\mathbf{15})\}^1,$$

where the predeterminated decimal minterms 7, 11, 13, 14 i 15 are given in bold.

The obtained minimal STF Y^1 corresponds to the minimal SOP (Sum-Of-Product expression) with the nonessential variable x_3, i.e. $f(x_1,x_2,-,x_4) = x_2x_4 \vee x_1x_4 \vee x_1x_2$.

If the submatrix Y^1 of the matrix M_4^3 of the same function f is covered by the conjuncterms-copies, the forced masks $\{lll-\}$ and $\{l-ll\}$ or by the masks $\{-lll\}$ and $\{l-ll\}$, we will have the following sets:

$$\overset{C}{\Rightarrow} \{(010-),(110-),(1-01)\}^1 \Rightarrow \{(-10-),(1-01)\}^1$$

$$\text{Or } \overset{C}{\Rightarrow}\{(-101),(-100),(1-01)\}^1 \Rightarrow \{(-10-),(1-01)\}^1.$$

By splitting the conjuncterm of the 3-rank $(1-01)$ using the matrix M_4^2, we will obtain:

$$\{(1-01)\}^1 \vdots \{(0-01),(0-10),(1-00)\}^0 \overset{S}{\Rightarrow}$$

$$\overset{S}{\Rightarrow}\begin{bmatrix} l-l- \\ l--l \\ --ll \end{bmatrix} = \left[\begin{array}{c:ccc} 1-0- & 0-0- & 0-1- & 1-0- \\ \underline{\underline{1--1}} & 0--1 & 0--0 & 1--0 \\ --01 & --01 & --10 & --00 \end{array}\right] \overset{C}{\Rightarrow} \{1--1\}^1.$$

Thus, the given function f has a minimal STF $Y^1 = \{(-10-),(1--1)\}^1$, where all variables are nonessential that corresponds to the minimal SOP $f(x_1,x_2,x_3,x_4) = x_2\bar{x}_3 \vee x_1x_4$. As one can see, the realization cost of the minimazed function is less, than those of the function with nonessential variable.

Among the weakly determinated functions, mainly occur those having so-called apparent variables: with minimal predeterminationof a function such variables manifest themselves as nonessential, and with the expansion of predetermination area of the same function as essential ones. Such properties of the weakly determinated functions are convenient to identify with the proposed method. Let us consider this in more detail.

If the function has an essential variable, it means that there is a pair of adjancent minterms, different by such a variable. In this case the splitting matrix M_n^{n-1} contains a pair of identical elements formed by the corresponding mask of literals: there are a two conjuncterms-copies of $(n-1)$-rank– one is in the submatrix Y^1 and the second is in the submatrix Y^0. Accordingly, if the matrix M_n^{n-1} does not contain such pairs, it means that each element of its submatrix Y^1 (or Y^0) has a equivalent in the subset $Y^{\sim}$. Therefore, if this function is predeterminated by minterms of the subset $Y^{\sim}$, then the matrix M_n^{n-1} could be covered by all its rows. This could correspond to the degenerated function where all variables are nonessentials. However, in fact, in these functions there is some subset of apparent nonessential variables, and the rest of them are essentials. To determine which variables are essentials and which are not, it is enough to expand the domain of the function predetermination by formation of the splitted conjuncterms of lower ranks using the matrices $M_n^{n-2}, M_n^{n-3},\ldots$. If on the *i*-stage of the splitting procedure the submatrix Y^1 of the matrix M_n^{n-i} has a row (or several rows) of elements that do not have any copies in the submatrix Y^0, then according to the Theorem, this function has one or more nonessential variables. The following execution of the splitting procedure on (in $(i+1)$–, $(i+2)$–, ..., stages) is meaningless, since in each row of the matrices M_n^{n-i+1}, M_n^{n-i+2}, ... the pairs of identical elements (one by one) will be formed in their submatrices Y^1 and Y^0.

The following examples illustrate the solution of this problem using the proposed method.

Example 3. To determine nonessential variables with the splitting method in the weakly determinated function $f(x_1, x_2, x_3, x_4)$ given in the perfect STF $\begin{cases} Y^1 = \{6,9\}^1 \\ Y^0 = \{5,12\}^0 \end{cases}$ (*this function is borrowed from* [3, *example* 11.2.2, p. 121]).

Solution. Let us perform the splitting procedure of the given minterms using the matrix M_4^3:

$$Y^1 \vdots Y^0 = \{(0110),(1001)\}^1 \vdots \{(0101),(1100)\}^0 \overset{s}{\Rightarrow} \begin{Bmatrix} lll- \\ ll-l \\ l-ll \\ -lll \end{Bmatrix} = \begin{bmatrix} 011- & 100- & \vdots & 010- & 110- \\ 01-0 & 10-1 & \vdots & 01-1 & 11-0 \\ 0-10 & 1-01 & \vdots & 0-01 & 1-00 \\ -110 & -001 & \vdots & -101 & -100 \end{bmatrix}.$$

As one can see, the matrix M_4^3 does not contain pairs of identical elements in one Y^1 and Y^0. This indicates lack of nonessential variables on the stage of the minimal predetermination of the function *f*. The continuation of the splitting procedure of the given minterms using the matrix M_4^2 gives the following:

$$\overset{s}{\Rightarrow} \begin{Bmatrix} ll-- \\ l-l- \\ l--l \\ -ll- \\ -l-l \\ --ll \end{Bmatrix} = \begin{bmatrix} 01-- & 10-- & \vdots & 01-- & 11-- \\ 0-1- & 1-0- & \vdots & 0-0- & 1-0- \\ \underline{\underline{0--0}} & \underline{\underline{1--1}} & \vdots & 0--1 & 1--0 \\ \underline{\underline{-11-}} & \underline{\underline{-00-}} & \vdots & -10- & -10- \\ -1-0 & -0-1 & \vdots & -1-1 & -1-0 \\ --10 & --01 & \vdots & --01 & --00 \end{bmatrix} \overset{c}{\Rightarrow} \begin{cases} 1.\ \{(0--0),(1--1)\}^1 \\ 2.\ \{(-11-),(-00-)\}^1 \end{cases}.$$

As the submatrix Y^1 has two rows of elements for the masks $\{l--l\}$ and $\{-ll-\}$ that do not have copies in the submatrix Y^0, then, according to the Theorem, the nonessential variables of the given function *f* are: for 1) x_2 and x_3, for 2) x_1 and x_4, that corresponds to [3].

For the solution 2) the minimal form can be found:

$$\{(-11-),(-00-)\}^1 \vdots \{(-10-)\}^0 \overset{s}{\Rightarrow} \begin{Bmatrix} -l-- \\ --l- \end{Bmatrix} = \begin{bmatrix} -1-- & \underline{\underline{-0--}} & \vdots & -1-- \\ \underline{\underline{--1-}} & --0- & \vdots & --0- \end{bmatrix} \overset{c}{\Rightarrow} \{(-0--),(--1-)\}^1.$$

Thus, the given function *f* has two solutions that reflect the minimal STF

$$\overset{c}{\Rightarrow} \begin{cases} 1.\ \{(0--0),(1--1)\}^1 \equiv \{(\mathbf{0,2,4,}6),(9,\mathbf{11,13,15})\}^1 \\ 2.\ \{(-0--),(--1-)\}^1 \equiv \{(\mathbf{0,1,2,3,8,}9,\mathbf{10,11}),(\mathbf{2,3,}6,\mathbf{7,10,11,14,15})\}^1 \end{cases},$$

where the predeterminated decimal minterms are highlighted in bold.

These solutions correspond to the analytical expressions: $\begin{cases} 1.\ f(x_1,-,-,x_4) = \bar{x}_1\bar{x}_4 \vee x_1x_4 \\ 2.\ f(-,x_2,x_3,-) = \bar{x}_2 \vee x_3 \end{cases}$.

Example 4. To determine nonessential variables with the splitting method in the weakly determinated function $f(x_1, x_2, x_3, x_4, x_5, x_6)$ given in the perfect STF $\begin{cases} Y^1 = \{(001010),(101111),(111101)\}^1 \\ Y^0 = \{(011001),(100011),(010110)\}^0 \end{cases}$ (*this function is borrowed from* [1, p.271]).

Solution. Let us perform the splitting procedure of the given minterms using the matrix M_6^5:

$$Y^1 \vdots Y^0 = \{(001010),(101111),(111101)\}^1 \vdots \{(011001),(100011),(010110)\}^0 \overset{S}{\Rightarrow}$$

$$\overset{S}{\Rightarrow} \begin{Bmatrix} lllll- \\ llll-l \\ lll-ll \\ ll-lll \\ l-llll \\ -lllll \end{Bmatrix} = \left[\begin{array}{ccc:ccc} 00101- & 10111- & 11110- & 01100- & 10001- & 01011- \\ 0010-0 & 1011-1 & 1111-1 & 0110-1 & 1000-1 & 0101-0 \\ 001-10 & 101-11 & 11-01 & 011-01 & 100-11 & 010-10 \\ 00-010 & 10-111 & 11-101 & 01-001 & 10-011 & 01-110 \\ 0-1010 & 1-1111 & 1-1101 & 0-1001 & 1-0011 & 0-0110 \\ -01010 & -01111 & -11101 & -11001 & -00011 & -10110 \end{array} \right].$$

As in the previous example, all variables are nonessential on the predetermination minimum stage of the given function *f*. However, if the given minterms are further split, starting with M_6^4, M_6^3, and etc., we obtain the following result for the matrix M_6^2:

$$\overset{S}{\Rightarrow} \begin{Bmatrix} ll---- \\ l-l--- \\ l--l-- \\ l---l- \\ l----l \\ -ll--- \\ -l-l-- \\ -l--l- \\ -l---l \\ --ll-- \\ --l-l- \\ --l--l \\ ---ll- \\ ---l-l \\ ----ll \end{Bmatrix} = \left[\begin{array}{ccc:ccc} 00---- & 10---- & 11---- & 01---- & 10---- & 01---- \\ 0-1--- & 1-1--- & 1-1--- & 0-1--- & 1-0--- & 0-0--- \\ 0--0-- & 1--1-- & 1--1-- & 0--0-- & 1--0-- & 0--1-- \\ 0---1- & 1---1- & 1---0- & 0---0- & 1---1- & 0---1- \\ 0----0 & 1----1 & 1----1 & 0----1 & 1----1 & 0----0 \\ -01--- & -01--- & -11--- & -11--- & -10--- & -10--- \\ -0-0-- & -0-1-- & -1-1-- & -1-0-- & -1-0-- & -1-1-- \\ -0--1- & -0--1- & -1--0- & -1--0- & -1--0- & -1--1- \\ -0---0 & -0---1 & -1---1 & -1---1 & -1---1 & -1---0 \\ --10-- & --11-- & --11-- & --10-- & --00-- & --01-- \\ --1-1- & --1-1- & --1-0- & --1-0- & --0-0- & --0-1- \\ --1--0 & --1--1 & --1--1 & --1--1 & --0--1 & --0--0 \\ ---01- & ---11- & ---10- & ---00- & ---00- & ---11- \\ \underline{\underline{---0-0}} & \underline{---1-1} & \underline{---1-1} & ---0-1 & ---0-1 & ---1-0 \\ ----10 & ----11 & ----01 & ----01 & ----11 & ----10 \end{array} \right] \overset{C}{\Rightarrow}$$

$$\overset{C}{\Rightarrow} \{(---0-0),(---1-1)\}^1.$$

From here we obtain $f(-,-,-,x_4,-,x_6) = \bar{x}_4\bar{x}_6 \vee x_4 x_6$, where x_1, x_2, x_3, x_5 are the nonessential variables, that corresponds to [1].

Example 5. To determine nonessential variables with the splitting method in the weakly determinated function $F(a,b,c,d,e,f,g,h)$ given in the Table 1.1 (*this function is borrowed from* [3, p. 134]).

Table 1.1.

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*F*
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	1	0	0
0	0	1	0	0	1	1	0	0
0	0	0	1	0	0	1	0	1
0	1	0	0	1	1	0	0	1
1	0	0	1	0	1	0	0	1

Solution. Let us perform the splitting procedure of the given minterms using the matrix M_8^7:

$$Y^1 \vdots Y^0 = \{(00010010),(01001100),(10010100)\}^1 \vdots \{(00100000),(01000010),(00100110)\}^0 \overset{S}{\Rightarrow}$$

$$\overset{S}{\Rightarrow}\left\{\begin{array}{l} lllllll- \\ llllll-l \\ lllll-ll \\ llll-lll \\ lll-llll \\ ll-lllll \\ l-llllll \\ -lllllll \end{array}\right\}=\left[\begin{array}{ccc|ccc}
0001001- & 0100110- & 1001010- & 0010000- & 0100001- & 0010011- \\
000100-0 & 010011-0 & 100101-0 & 001000-0 & 010000-0 & 001001-0 \\
00010-10 & 01001-00 & 10010-00 & 00100-00 & 01000-10 & 00100-10 \\
0001-010 & 0100-100 & 1001-100 & 0010-000 & 0100-010 & 0010-110 \\
000-0010 & 010-1100 & 100-0100 & 001-0000 & 010-0010 & 001-0110 \\
00-10010 & 01-01100 & 10-10100 & 00-00000 & 01-00010 & 00-00110 \\
0-010010 & 0-001100 & 1-010100 & 0-100000 & 0-000010 & 0-100110 \\
-0010010 & -1001100 & -0010100 & -0100000 & -1000010 & -0100110
\end{array}\right].$$

The given function *f*, as in the examples 3 and 4, does not contain any essential variable on the stage of minimum predetermination. By expanding the area of predetermination of this function *f* using matrices M_8^6, M_8^5, M_8^4, etc., we obtain the result for the mask $\{---ll---\}$ of the matrix M_8^1:

$$\overset{S}{\Rightarrow}\left\{\begin{array}{l} ll------ \\ l-l----- \\ l--l---- \\ l---l--- \\ l----l-- \\ l-----l- \\ l------l \\ -ll----- \\ -l-l---- \\ -l--l--- \\ -l---l-- \\ -l----l- \\ -l-----l \\ --ll---- \\ --l-l--- \\ --l--l-- \\ --l---l- \\ --l----l \\ ---ll--- \\ ---l-l-- \\ ---l--l- \\ ---l---l \\ ----ll-- \\ ----l-l- \\ ----l--l \\ -----ll- \\ -----l-l \\ ------ll \end{array}\right\}=\left[\begin{array}{ccc|ccc}
00------ & 01------ & 10------ & 00------ & 01------ & 00------ \\
0-0----- & 0-0----- & 1-0----- & 0-1----- & 0-0----- & 0-1----- \\
0--1---- & 0--0---- & 1--0---- & 0--0---- & 0--0---- & 0--0---- \\
0---0--- & 0---1--- & 1---0--- & 0---0--- & 0---0--- & 0---0--- \\
0----0-- & 0----1-- & 1----0-- & 0----0-- & 0----0-- & 0----1-- \\
0-----1- & 0-----0- & 1-----0- & 0-----0- & 0-----1- & 0-----1- \\
0------0 & 0------0 & 1------0 & 0------0 & 0------0 & 0------0 \\
-00----- & -10----- & -00----- & -01----- & -10----- & -01----- \\
-0-1---- & -1-0---- & -0-0---- & -0-0---- & -1-0---- & -0-0---- \\
-0--0--- & -1--1--- & -0--0--- & -0--0--- & -1--0--- & -0--0--- \\
-0---0-- & -1---1-- & -0---0-- & -0---0-- & -1---0-- & -0---1-- \\
-0----1- & -1----0- & -0----0- & -0----0- & -1----1- & -0----1- \\
-0-----0 & -1-----0 & -0-----0 & -0-----0 & -1-----0 & -0-----0 \\
--01---- & --00---- & --01---- & --10---- & --00---- & --10---- \\
--0-0--- & --0-1--- & --0-0--- & --1-0--- & --0-0--- & --1-0--- \\
--0--0-- & --0--1-- & --0--1-- & --1--0-- & --0--0-- & --1--1-- \\
--0---1- & --0---0- & --0---0- & --1---0- & --0---1- & --1---1- \\
--0----0 & --0----0 & --0----0 & --1----0 & --0----0 & --1----0 \\
\underline{\underline{---10---}} & \underline{\underline{---01---}} & \underline{\underline{---10---}} & ---00--- & ---00--- & ---00--- \\
---1-0-- & ---0-1-- & ---1-1-- & ---0-0-- & ---0-0-- & ---0-1-- \\
---1--1- & ---0--0- & ---1--0- & ---0--0- & ---0--1- & ---0--1- \\
---1---0 & ---0---0 & ---1---0 & ---0---0 & ---0---0 & ---0---0 \\
----00-- & ----11-- & ----01-- & ----00-- & ----00-- & ----01-- \\
----0-1- & ----1-0- & ----0-0- & ----0-0- & ----0-1- & ----0-1- \\
----0--0 & ----1--0 & ----0--0 & ----0--0 & ----0--0 & ----0--0 \\
-----01- & -----10- & -----10- & -----00- & -----01- & -----11- \\
-----0-0 & -----1-0 & -----1-0 & -----0-0 & -----0-0 & -----1-0 \\
------10 & ------00 & ------00 & ------00 & ------10 & ------10
\end{array}\right]\overset{C}{\Rightarrow}$$

$$\overset{C}{\Rightarrow}\{(---10---),(---01---)\}^1.$$

Answer. Thus, the given function $F(-,-,-,d,e,-,-,-)=d\bar{e}\vee\bar{d}e$, where *a*, *b*, *c*, *f*, *g*, *h* are the nonessential variables, that corresponds to [3, p. 174].

4. **Effectiveness of the proposed method**. The comparison of the proposed method with the known methods demonstrates the following advanced features of its implementation. If the mask of literals of the certain *r*-rank is imposed on an even number of *k* minterms of the given function *f*, then in formed splitting matrix M_n^r one can immediately (without any additional operations or procedures) determine one nonessential variable for $r=n-1$, or more nonessential variables if $r<n-1$. This is proved by the described above Theorem and its corollary (*see* Section 2).

This fundamentally differentiates the proposed method from the known methods [1–8], where more implementation steps are required. In particular, when comparing with the tabulated methods [3–7] and analytical methods [3, 6, 8], the advantages of the proposed method are obvious. For example, as mentioned in [3, p. 122, example 11.3.1], to answer the question whether a given the weakly determinated function with $n = 4$ has any nonessential variables or not, it is necessary to perform 5 steps of the algorithm. Meanwhile, the proposed algorithm requires only 2 steps: the splitting procedure of the given minterms and the verification of Theorem (Section 2) on the covering of the splitting matrix M_n^{n-1} Algorithms of the heuristic methods [1, 4, 8] as they are based on the Shannon expansion for identification of a nonessential variable involve having two adjacent minterms and for identification of two nonessential variables – four adjacent minterms, to identify three nonessential variables – eight adjacent minterms, etc.. Similarly, the algorithm based on the functional decomposition [7, 8] and the decomposition clones is very complicated [9]. That is why, in this paper we have borrowed the examples from the well-known publications for comparison of different approach and illustration of the advantages of the presented method.

The simplicity of the implementation of the proposed algorithm is particularly evident for the case of determination of nonessential variables in the incomplete and especially in the weakly determinated functions. Since the splitting matrix M_n^r consists of a basic (for Y^1) and additional matrices (for $Y^{\sim}$ if the function *f* is inpredeterminated, or for Y^0 if the function *f* is weakly determinated), the simultaneous execution of procedures of predetermination of the incomplete functions is provided. This significantly simplifies the algorithm of search of nonessential variables.

Conclusion

A new method for the reduction of the number of variables in complete and incomplete logic functions by eliminating nonessential variables is described. This method is based on the splitting conjuncterms procedure, which in comparison with the known algorithms provides a relatively simpler implementation.

1. *Zakrevskij A.D.*, *Pottosin Ju.V.*, *Cheremisinova L.D.* Logicheskie osnovy proektirovanija diskretnyh ustrojstv – M.: Fizmatlit, 2007. – P. 269–277. (In Russian).
2. *Kambayashi Y.* Logic design of programmable logic arrays // IEEE Trans. on Computers, **C-28**, Sept. 1979. – N 9. – P. 609–617.
3. *Sasao T.* Memory-Based Logic Synthesis. – Springer Science+Business Media, LLC 2011. – P. 119–135. – www.springer.com/us/book/9781441981035
4. *Implicit* algorithm for multi-valued input support manipulation / A. Mishchenko, C. Files, M. Perkowski et al. // Proc. 4thIntl. Workshop on Boolean Problems, Sept. 2000, Freiberg, Germany.
5. *Sasao T.* On the number of dependent variables for incompletely specified multiple-valued functions // 30th Int. Symp. Multiple-Valued Logic, Portland, Oregon, U.S.A., May 23–25, 2000. – P. 91–97. – http://dblp.uni-trier.de/db/conf/ismvl/ismvl2000.html#Sasao00
6. *Majewski W.* Uklady logiczne. Wybrane zagadnienia. – Warszawa, WPW. – 1997. – 180 s.
7. *Sasao T.* On the number of variables to represent sparse logic functions, ICCAD-2008, San Jose, CA, USA, Nov. 10–13, 2008. – P. 45–51. – http://alcom.ee.ntu.edu.tw/system/privatezone/meetingfile/200903100035361.pdf
8. *Sasao T.* On the number of LUTs to realize sparse logic functions, Int. Workshop on Logic and Synthesis (IWLS-2009), Berkeley, CA, U.S.A., July 31 – Aug. 2, 2009. – P. 64–71. – http://www.lsi-cad.com/sasao/Papers/files/ IWLS2009_ sasao.pdf
9. *Rytsar B.* Reduktsiya nesuttyevykh zminnykh systemy bul'ovykh funktsiy // Visn. NU Radioelektronika i telekomunikatsiyi. – 2001. – N 428. – P. 184–189. (In Ukrainian).
10. *Rytsar B.Ye.* Teoretyko-mnozhynni optymizatsiyni metody lohichnoho syntezu kombinatsiynykh merezh: Dys. … d.t.n. – L'viv, 2004. – 348 p. (In Ukrainian).
11. *Rytsar B.*, *Romanowski P.*, *Shvay A.* Set-theoretical Constructions of Boolean Functions and theirs Applications in Logic Synthesis // Fundamenta Informaticae. – 2010. – **99**, N 3. – P. 339–354.

Поступила 21.05.2017
E-mail: *bohdanrytsar@gmail.com*

B.Ye. Rytsar– Doctor Sc., Professor, Department of Radioelectronic Devices Systems, Institute of Telecommunications, Radioelectronics and Electronic Engineering, L'viv polytechnic National University, Ad.: Bandera srt., 12, L'viv, Ukraine, e-mail: bohdanrytsar@gmail.com

●